NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL: 6**
COURSE CODE: MAP602S	**COURSE NAME:** Mathematical Programming
SESSION: January 2019	**PAPER:** Theory
DURATION: 3 Hours	**MARKS:** 85

SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINERS	MR. B.E OBABUEKI MRS. S. MWEWA
MODERATOR:	DR. A.S EEGUNJOBI

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.
2. **Graph papers to be supplied by Examinations Department**

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Excluding this front page)

Question 1 (12 marks)

John has $20,000 which he must invest in three funds F1, F2 and F3. Fund F1 offers a return of 2% and has a low risk. Fund F2 offers a return of 4% and has a medium risk. Fund F3 offers a return of 5% but has a high risk. To be on the safe side, John invests no more than $3000 in F3 and at least twice as much as in F1 than in F2. The rates hold till the end of the year. John wants to know what amounts he should invest in each fund to maximize the year end return.

(http://www.analyzemath.com/linear_programming/linear_prog_applications.html)

Model this linear problem. You must declare your variables and identify your constraints unambiguously. (12)

Question 2 (8 marks)

Consider the following linear program:

$$
\begin{array}{lll}
\textit{Maximize} & m = 11x + 11y & \\
\textit{Subject to} & 5x + 3y \leq 15 & \textit{sugar quantity} \\
 & 3x + 4y \leq 12 & \textit{salt quantity} \\
 & x + 5y \geq 5 & \textit{vitamine A quantity} \\
 & x, y \geq 0 &
\end{array}
$$

Explain in detail how you would obtain the dual price and allowable increase of sugar if the exact amount of sugar required was 15. (8)

Question 3 (24 marks)

Consider the following linear program:

$$
\begin{array}{ll}
\textit{Minimize} & m = 30x + 24y \\
\textit{Subject to} & 5x + 3y \geq 15 \\
 & 3x + 4y \geq 12 \\
 & x, y \geq 0
\end{array}
$$

3.1 Write down the dual of this linear program. (5)

3.2 Solve the dual of the linear program using graphical method. (10)

3.3 Use the solution of the dual to obtain the solution of the primal model. (9)

Question 4 (25 marks)

Consider the linear program:

$$\begin{aligned} &\textit{Maximize} \quad P = 12x + 15y + 9z \\ &\textit{Subject to} \quad 8x + 16y + 12z \le 250 \\ &\qquad\qquad\quad 4x + 8y + 10z \ge 80 \\ &\qquad\qquad\quad 7x + 9y + 8z = 105 \\ &\qquad\qquad\quad x, y, z \ge 0 \end{aligned}$$

(http://www.universalteacherpublications.com/univ/ebooks/or/Ch3/twophase1.htm)

4.1 Develop the objective function for phase 1 of the two-phase method. (2)

4.2 Determine the final tableau of phase 1. You must show all the tableaux leading to this final tableau. (9)

4.3 Assume that the final tableau of phase 1 is

$$\begin{matrix} x & y & z & s_1 & s_2 & A_2 & A_3 & H & \\ \end{matrix}$$
$$\begin{pmatrix} 0 & 400 & 0 & 95 & 50 & -50 & -80 & 0 & 11350 \\ 0 & 100 & 190 & 0 & -35 & 35 & -20 & 0 & 700 \\ 19 & 13 & 0 & 0 & 4 & -4 & 5 & 0 & 205 \\ 0 & 0 & 0 & 0 & 0 & -5 & -5 & 5 & 0 \end{pmatrix}.$$

Use this tableau to develop the objective function for phase 2 of the two-phase method. (5)

4.4 One of the tableaux in phase 2 is

$$\begin{matrix} x & y & z & s_1 & s_2 & P & \\ \end{matrix}$$
$$\begin{pmatrix} 0 & 400 & 0 & 95 & 50 & 0 & 11350 \\ 0 & 100 & 190 & 0 & -35 & 0 & 700 \\ 19 & 13 & 0 & 0 & 4 & 0 & 205 \\ 0 & -1560 & 0 & 0 & 660 & 760 & 123600 \end{pmatrix}$$

Improve on this tableau to get the optimal solution of the original linear program. (9)

Question 5 (16 marks)

Four jobs (J1, J2, J3, and J4) need to be executed by four workers (W1, W2, W3, and W4), one job per worker. The matrix below shows the cost of assigning a certain worker to a certain job.

Job	Worker			
	W1	W2	W3	W4
J1	82	83	69	92
J2	77	37	49	92
J3	11	69	5	86
J4	8	9	98	23

http://www.hungarianalgorithm.com/examplehungarianalgorithm.php

Use the Hungarian algorithm to assign the jobs to the workers to minimize the total cost.

(16)

END OF PAPER **TOTAL MARKS: 85**